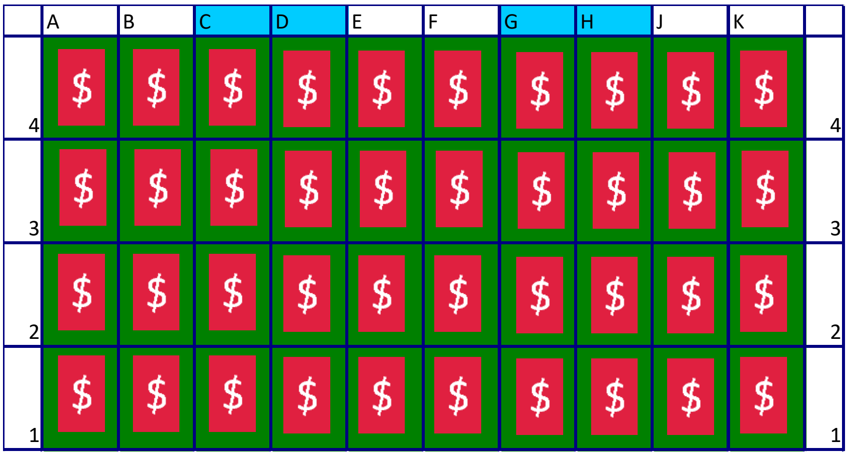

Summary

Rank values found in Stratego literature stem from experience and common sense. This document shows an attempt to give a theoretical basis to rank values. In the Gravon database it is possible to count the number of captures by each rank in all games. These counts can be transformed to a system of linear equations. The solving of this system of linear equations makes it possible to determine the ratio of rank values relative to the value of the marshal. A standard calibration value of 100 for the marshal enables the comparison of the rank values found here with rank values in Stratego literature.

Contents

1 Rank value in literature

1.1 Why static rank values?

Stratego programs contain functions that somehow determine the best move in a position. The most used mechanism is to compute the value of each possible move followed by the choice of the move with the best value. Literature mostly mentions this mechanism in programs that apply a minimax algorithm, but also other mechanisms implement the selection of the best move by values given to the possible moves.
The determination of the value of a move often depends on values assigned to the pieces of a rank. Usually the value of a rank consist of two components:

- A fixed basic value that each piece of a rank has in the initial position
- Corrections to this basic value if specific conditions are present in a game position.

Literature mentions a number of valuation scales with fixed values for ranks.
The basic values of these value scales show differences. Besides authors of these valuation scales define specific corrections for conditions that may occur in game positions.

In order to make the valuation scales of different authors comparable in the following overview a considerable amount of the nuances in the original scales have been omitted.

Rank	JM	KS	MS	SA	VB
Marshal	100	100	100	100	100
General	50	90	75	50	86
Colonel	20	58	44	25	59
Major	10	37	35	19	41
Captain	4	26	25	12½	28
Lieutenant	2	21	12½	6¼	20
Sergeant	1	16	5	4	13
Miner	10	32	25	6¼	9
Scout	½	18	2½	7½	6
Spy	20	42	25	50	50
Bomb	150	26	19	5	50
Flag	200	316	250	2500	100

All scales show a gradual decrease of the rank values from marshal to sergeant. This trend is not present in the ranks of miner, scout, spy, bomb and flag.

Reference	Description
JM	Jeroen Mets, Monte Carlo Stratego, 2008
KS	Karl Stengård, Utveckling av minimax-baserad agent för strategispelet Stratego, 2006
MS	Maarten P.E. Schadd en Mark H.M. Winands, Quiescence Search for Stratego, 2009
SA	Sander Arts, Competitive play in Stratego, 2010
VB	Vincent de Boer: Invincible, A Stratego bot, 2007

1.2 Specific conditions and value corrections

1.2.1 Rank value changes during a game

Valuation scales contain static values for ranks. The term static supposes that static values should be applicable during all phases of a game. But the capability of pieces to capture material diminishes during a game because less and less pieces are available for capture. Because of this fact the values in valuation scales should be considered to be averages: at the start of a game the real capability to capture material is greater than at the end of a game. Literature mentions corrections to the basic values taking into account the disappearance of a rank from the board as well as corrections because of the number of pieces of a rank on the board.

1.2.2 Position

For various reasons a piece can get additional value by its position on the board. Authors name position corrections as a means to stimulate the reduction of distance between a strong and weak piece. Control of the lanes may also be a reason the assign additional value to a piece.

1.2.3 Too many bombs known

A special condition applies for bombs. If 5 or 6 bombs are known, the chance of loss by capture of the flag increases considerably.

1.2.4 Invulnerable ranks

If a rank cannot be captured by another rank, the value of pieces with this rank is higher than when this rank can be captured. This kind of invulnerability may also occur on a part of the board during a limited amount of moves, if many pieces are present on the board. If the lanes are mutually unattainable, the detection of a top rank in a lane can result in the local and temporary invulnerability of a less strong rank in other lanes.

1.2.5 The miner and scout in the end game

Miners get additional value, if the location of bombs around the flag becomes known. The chances of occurrence are higher in the end of the game.
In the endgame the availability of a scout can enforce a draw despite large material disadvantage.

1.2.6 The value for the capture of the flag

By capture of the flag a player wins the game. Valuation scales in literature give a value to the flag in such a way that an attack to flag will always have a higher value than any other move value. The only purpose of an extremely high value is to ensure that a capture of the flag will be preferred above any other high valued moves.
The flag is not able to capture other ranks, so the flag value cannot represent value won by captures. Therefore the flag is excluded from computations of the static value.

2 The investigation of static rank value

2.1 Need for a theoretical base

In games like chess the average static value of pieces can be established from knowledge of game theory. That kind of knowledge is missing in Stratego. Some authors mention the lack of a theoretical base and see that lack as an extra barrier for the design and creation of a Stratego program. From sheer necessity anybody who wants to implement static values in a Stratego program has to devise a valuation scale himself. This is however just part of the job. Corrections are necessary for special conditions. No other way remains till this moment as using the trial and error method for the determination of extra rules which at the end will realize acceptable results.

This study is an attempt to improve and refine this situation. The Gravon website offers the possibility to download data of games played on Gravon (http://www.gravon.de/strados2/files/).
The author has made the choice to select a snapshot of games played form May 2003 to June 2006. Games from later periods have not been selected in order to prevent the inclusion of games between human players and Stratego bots as much as possible.
The games in this snapshot show how many pieces of a rank have captured pieces of other ranks. This enables to ground a rank value on the winning capability of a rank.

2.2 Material and detection value

The ranks from marshal to bomb get their value by their capability to capture pieces. That capability has a value equal to the value of pieces that have been captured by pieces of the rank.
A consequence is that the flag according to this definition gets value = 0.

A different kind of value arises by information about the rank of a piece. Both Jeroen Mets and Vincent de Boer mention that the detection of a rank generates value. They give values for the detection of ranks without a theoretical underpinning.

Condition	Generates value
A known piece attacks and wins	Material: the attacked piece
An unknown piece attacks and wins	Material: the attacked piece Detection: part of attacking piece
A known piece has been attacked and wins	Material: the attacking piece
An unknown piece has been attacked and wins	Material: the attacking piece Detection: part of attacked piece

This study reveals how rank values vary by varying the factor for detection of a rank from 0 (the detection does not generate value) to 1(the detection makes the piece worthless).
Other kinds of value have not been examined.

2.3 Analysis and results

2.3.1 Model 1: all ranks have the same detection factor

First computations have been made with the assumption that the detection factor is the same for all ranks.

Model 1: all ranks have equal detection factors

	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Detection factor	0	0	0	0	0	0	0	0	0	0	0
	100	106	85	64	48	41	49	164	4	72	340
Detection factor	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
	100	106	85	65	51	44	48	114	20	83	214
Detection factor	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2
	100	106	84	66	53	47	48	80	30	92	126
Detection factor	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3
	100	105	84	66	54	49	49	55	37	98	60
Detection factor	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4
	100	104	83	66	55	50	50	37	42	103	9

Questionable however is whether the detection is equal for all ranks. The study of Jeroen Mets (Monte Carlo Stratego, 2008) shows different values for the ranks:

Rank	Material	Detection
Marshal	100	0,8
General	50	0,8
Colonel	20	0,75
Major	10	0,6
Captain	4	0,25
Lieutenant	2	0
Sergeant	1	0
Miner	10	0,6
Scout	½	0
Spy	20	0
Bomb	150	0,27

A theoretical base for these values is missing. The question here arises what the right detection factor should be per rank. At this moment hard clues are missing. Then nothing else can be done than a reasoned choice of values. In this study the choice has been made to give higher detection factors to spy and bomb. That choice has been worked out in paragraph 2.3.2.

In a subsequent attempt the choice has been made to assign a detection factor 0 to the ranks captain to sergeant and the scout. That choice has been worked out in paragraph 2.3.3.

2.3.2 Model 2: Bomb and spy have a higher detection factor

Bomb and spy get a higher detection factor than the other ranks.
The underlying idea is that bomb and spy lose the capability to win material in a greater extent than other ranks.

Model 2: bombs have a large detection factor, other ranks have equal detection factors

	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Detection factor	0	0	0	0	0	0	0	0	0	0	0
	100	106	85	64	48	41	49	164	4	72	340
Detection factor	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,25	0,25
	100	104	83	64	50	44	46	94	24	86	170
Detection factor	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,2	0,5	0,5
	100	101	81	63	51	45	45	61	33	93	85
Detection factor	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,3	0,75	0,75
	100	98	78	62	50	45	45	42	37	97	35
Detection factor	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	0,4	1	1
	100	94	75	60	50	45	44	31	38	100	2

2.3.3 Model 3: Captain to sergeant and scout have detection factor = 0

Bomb, and spy get a higher detection factor than the other ranks. In addition the ranks from captain to sergeant and scout get a detection factor = 0. Miners get the same detection factor as strong ranks. The underlying idea is that it is highly impossible gain significant advantage by the detection of these ranks.

2.3.4 Trends in these models

A general trend is that the capability to gain value decreases gradually from marshal to sergeant.

Models with detection factor 0 of a low detection factor show that bombs are the real killers in Stratego. An possible but surprising cause must be that in many games high quality ranks run on bombs by a blind attack.
Miners derive their value from bombs, since they are the only rank able to capture these killers.

Models with a low detection value have absurdly high rank values for bomb and miner. The value of bombs and miners strongly decrease with the increase of the detection factor. That emphasizes the importance that stems from the detection of a rank.

For high detection factors the value of the scout increase strongly. That exemplifies the special function the scout has in Stratego.

2.4 Comparison with literature

A comparison shows that rank values in this study hardly correspond to rank values in literature.

Rank	JM	KS	MS	SA	VB	Study
Marshal	100	100	100	100	100	100
General	50	90	75	50	86	95
Colonel	20	58	44	25	59	77
Major	10	37	35	19	41	61
Captain	4	26	25	12½	28	50
Lieutenant	2	21	12½	6¼	20	42
Sergeant	1	16	5	4	13	39
Miner	10	32	25	6¼	9	39
Scout	½	18	2½	7½	6	33
Spy	20	42	25	50	50	95
Bomb	150	26	19	5	50	42
Flag	200	316	250	2500	100	-

The best fit exist between model 3 with the highest detection factors and the rank value scale of Karl Stengård. With a few exceptions the valuation of ranks in literature is lower than in this study.

But in literature large differences exist between the values shown by the various authors too. That does not come as a surprise. Research in psychology has shown that in the process of estimation of chances people let themselves be guided mostly by knowledge and intuition based on experience. If hard information is missing in their experience people lack the capability to make a good estimation of chances.
This may be an explanation for the differences between the valuation systems of authors, but it does not explain the systematically higher values for general to bomb that in this study has been found.

This study is an attempt to measure what a rank really performs on the board by material and detection gain of value. In comparison with this study the importance of ranks below the marshal is being underestimated in literature.
The cause of this underestimation may lay in the fact that authors in literature have only considered the capability to capture other ranks and have disregarded the ability to generate value by the detection of other ranks.

3 The computation method

3.1 Overview of moves that produce material gain

The value of a rank is defined as the value of pieces that have produced material gain.
The value of these pieces of one rank equals to the sum of the values of captured pieces.

The flag cannot capture other pieces and therefore in accordance with this definition the value of the flag = 0, an argument to neglect moves to the flag totally in this study.

Each row shows the number of pieces and their winning attacks on a rank

Rank	NrPieces	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Marshal	46248	0	8277	10281	11601	11785	9809	8432	10119	17438	2242	0
General	42353	0	0	9988	12671	14027	11979	9438	10660	17004	3877	0
Colonel	70695	0	0	0	21437	22546	18438	15507	17273	25845	5823	0
Major	88644	0	0	0	0	33407	25714	20248	19965	31809	4894	0
Captain	103767	0	0	0	0	0	39357	28033	22272	45428	3367	0
Lieutenant	82780	0	0	0	0	0	0	28778	20913	47094	2249	0
Sergeant	59821	0	0	0	0	0	0	0	25979	38015	1718	0
Miner	66171	0	0	0	0	0	0	0	0	30550	2755	48813
Scout	2374	0	0	0	0	0	0	0	0	0	7052	0
Spy	9010	9010	0	0	0	0	0	0	0	0	0	0
Bomb	86020	3104	4294	6965	8891	11802	11254	8212	0	33092	1699	0

Example: the material value of 9010 spies is equal to the value of 9010 pieces with the rank of marshal and captured by the spy. At first this suggests that the value of the spy is equal to the value of the marshal. That is incorrect, because the marshal captures 2242 spies.

3.2 Overview of moves that produce detection value

The value of a rank is defined as the value of pieces that have produced detection value.
The value of these pieces of one rank equals to the sum of the values of winning pieces * detection factor.

Each row shows the number of pieces and their detections of a rank

Rank	NrPieces	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Marshal	11386	0	0	0	0	0	0	0	0	0	9010	2932
General	8425	5522	0	0	0	0	0	0	0	0	0	4122
Colonel	15993	6326	5906	0	0	0	0	0	0	0	0	6706
Major	27802	6222	6673	13821	0	0	0	0	0	0	0	8503
Captain	42977	5508	6670	13402	24128	0	0	0	0	0	0	11359
Lieutenant	43074	4417	5359	10153	17071	31207	0	0	0	0	0	10939
Sergeant	32392	3609	3771	7538	12073	20399	24326	0	0	0	0	7931
Miner	36822	3393	2901	6362	9770	13993	15986	23428	0	0	0	0
Scout	130851	11092	10086	16941	22962	36098	40889	35076	27692	0	0	31926
Spy	3479	159	987	2478	2640	2070	1579	1317	2350	2374	0	1602
Bomb	20383	0	0	0	0	0	0	0	36129	0	0	0

Example: 11386 marshals have been lost to the ranks spy and bomb. That has produced a detection value of 9010 spies and 2932 bombs.
The detection value of 9010 spies = the value of 9010 spies * the detection factor of the spy.
The detection value of 2932 bombs = the value of 2932 bombs * the detection factor of the bomb.

3.3 Linear equations for material gain

Back to the overview in paragraph 3.1.

Each row shows the number of pieces and their winning attacks on a rank												
Rank	NrPieces	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Marshal	46248	0	8277	10281	11601	11785	9809	8432	10119	17438	2242	0
General	42353	0	0	9988	12671	14027	11979	9438	10660	17004	3877	0
Colonel	70695	0	0	0	21437	22546	18438	15507	17273	25845	5823	0
Major	88644	0	0	0	0	33407	25714	20248	19965	31809	4894	0
Captain	103767	0	0	0	0	0	39357	28033	22272	45428	3367	0
Lieutenant	82780	0	0	0	0	0	0	28778	20913	47094	2249	0
Sergeant	59821	0	0	0	0	0	0	0	25979	38015	1718	0
Miner	66171	0	0	0	0	0	0	0	0	30550	2755	48813
Scout	2374	0	0	0	0	0	0	0	0	0	7052	0
Spy	9010	9010	0	0	0	0	0	0	0	0	0	0
Bomb	86020	3104	4294	6965	8891	11802	11254	8212	0	33092	1699	0

Each row in this overview can be expressed as a mathematical formula:

$$c^k * v^k = \sum_{i=Marshal\ (i \neq k)}^{Flag} c^i * v^i$$

The variable c is a constant that represents a count. At the left of the = sign that count is equal to is the count of pieces that have captured another piece, at the right that count is equal to the count of moves that capture another rank.
In this equation the k represents one of the ranks from marshal to bomb.
In the sum at the right of the = sign i goes from the rank of marshal to the rank of bomb and skips the rank = k, because a piece cannot gain material value by capturing the same rank.

The 11 equations together give a system of 11 equations with 11 unknown variables.
The term at the left of the = sign ($c^k * v^k$) can be transferred to the right.
By the transfer the overview changes to:

Each row shows the coefficients of winning attacks on a rank											
Rank	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Marshal	-46248	8277	10281	11601	11785	9809	8432	10119	17438	2242	0
General	0	-42353	9988	12671	14027	11979	9438	10660	17004	3877	0
Colonel	0	0	-70695	21437	22546	18438	15507	17273	25845	5823	0
Major	0	0	0	-88644	33407	25714	20248	19965	31809	4894	0
Captain	0	0	0	0	-103767	39357	28033	22272	45428	3367	0
Lieutenant	0	0	0	0	0	-82780	28778	20913	47094	2249	0
Sergeant	0	0	0	0	0	0	-59821	25979	38015	1718	0
Miner	0	0	0	0	0	0	0	-66171	30550	2755	48813
Scout	0	0	0	0	0	0	0	0	-2374	7052	0
Spy	9010	0	0	0	0	0	0	0	0	-9010	0
Bomb	3104	4294	6965	8891	11802	11254	8212	0	33092	1699	-86020

The transfer to the right leads to 11 equations with the formula:

$$0 = -c^k * v^k + \sum_{i=Marshal\ (i \neq k)}^{Flag} c^i * v^i$$

3.4 Linear equations for detection gain

Back to the overview in paragraph 3.2.

Each row shows the number of pieces and their detections of a rank											
Rank	NrPieces	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Marshal	11386	0	0	0	0	0	0	0	0	0	9010	2932
General	8425	5522	0	0	0	0	0	0	0	0	0	4122
Colonel	15993	6326	5906	0	0	0	0	0	0	0	0	6706
Major	27802	6222	6673	13821	0	0	0	0	0	0	0	8503
Captain	42977	5508	6670	13402	24128	0	0	0	0	0	0	11359
Lieutenant	43074	4417	5359	10153	17071	31207	0	0	0	0	0	10939
Sergeant	32392	3609	3771	7538	12073	20399	24326	0	0	0	0	7931
Miner	36822	3393	2901	6362	9770	13993	15986	23428	0	0	0	0
Scout	130851	11092	10086	16941	22962	36098	40889	35076	27692	0	0	31926
Spy	3479	159	987	2478	2640	2070	1579	1317	2350	2374	0	1602
Bomb	20383	0	0	0	0	0	0	0	36129	0	0	0

Each row in this overview can be expresses as a mathematical formula:

$$n^k * v^k = \sum_{i=Marshal\ (i \neq k)}^{Flag} n^i * d^i * v^i$$

The variable n is a constant that represents a count. At the left of the = sign that count is equal to is the count of pieces that have been captured by another piece, at the right that count is equal to the count of moves that detect another rank.
In this equation the k represents one of the ranks from marshal to bomb.
In the sum at the right of the = sign i goes from the rank of marshal to the rank of bomb and skips the rank = k, because a piece cannot gain detection value by capturing the same rank.
d^i is the for a rank specific detection factor with values from 0 to 1 in this study.

The 11 equations together give a system of 11 equations with 11 unknown variables.
The term at the left of the = sign ($n^k * v^k$) can be transferred to the right.
By the transfer the overview changes to:

Each row shows the coefficients of detections of a rank											
Rank	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Marshal	-11386	0	0	0	0	0	0	0	0	9010	2932
General	5522	-8425	0	0	0	0	0	0	0	0	4122
Colonel	6326	5906	-15993	0	0	0	0	0	0	0	6706
Major	6222	6673	13821	-27802	0	0	0	0	0	0	8503
Captain	5508	6670	13402	24128	-42977	0	0	0	0	0	11359
Lieutenant	4417	5359	10153	17071	31207	-43074	0	0	0	0	10939
Sergeant	3609	3771	7538	12073	20399	24326	-32392	0	0	0	7931
Miner	3393	2901	6362	9770	13993	15986	23428	-36822	0	0	0
Scout	11092	10086	16941	22962	36098	40889	35076	27692	-130851	0	31926
Spy	159	987	2478	2640	2070	1579	1317	2350	2374	-3479	1602
Bomb	0	0	0	0	0	0	0	36129	0	0	-20383

The transfer to the right leads to 11 equations with the formula:

$$0 = -n^k * v^k + \sum_{i=Marshal\ (i \neq k)}^{Flag} n^i * d^i * v^i$$

3.5 Linear equations for gain by both material and detection

The value that a rank can win is the sum of material and detection gain. This can be expressed in a formula by combining the formulas for material gain and detection gain.

$$c^k * v^k + n^k * v^k = \sum_{i=Marshal\ (i \neq k)}^{Flag} c^i * v^i + n^i * d^i * v^i$$

Here again the terms at the left can be transferred to the right:

$$0 = -c^k * v^k - n^k * v^k + \sum_{i=Marshal\ (i \neq k)}^{Flag} c^i * v^i + n^i * d^i * v^i$$

This formula contains the term $n^i * d^i * v^i$ that depends on the detection factor d^i of a rank. In this study the values of the detection factor have been varied as follows:

Rank	All detection factors equal				
Marshal	0	0,1	0,2	0,3	0,4
General	0	0,1	0,2	0,3	0,4
Colonel	0	0,1	0,2	0,3	0,4
Major	0	0,1	0,2	0,3	0,4
Captain	0	0,1	0,2	0,3	0,4
Lieutenant	0	0,1	0,2	0,3	0,4
Sergeant	0	0,1	0,2	0,3	0,4
Miner	0	0,1	0,2	0,3	0,4
Scout	0	0,1	0,2	0,3	0,4
Spy	0	0,1	0,2	0,3	0,4
Bomb	0	0,1	0,2	0,3	0,4

Rank	Detection factors of spy and bomb larger				
Marshal	0	0,1	0,2	0,3	0,4
General	0	0,1	0,2	0,3	0,4
Colonel	0	0,1	0,2	0,3	0,4
Major	0	0,1	0,2	0,3	0,4
Captain	0	0,1	0,2	0,3	0,4
Lieutenant	0	0,1	0,2	0,3	0,4
Sergeant	0	0,1	0,2	0,3	0,4
Miner	0	0,1	0,2	0,3	0,4
Scout	0	0,1	0,2	0,3	0,4
Spy	0	0,25	0,5	0,75	1
Bomb	0	0,25	0,5	0,75	1

Rank	Detection factors canon futter = 0				
Marshal	0	0,1	0,2	0,3	0,4
General	0	0,1	0,2	0,3	0,4
Colonel	0	0,1	0,2	0,3	0,4
Major	0	0,1	0,2	0,3	0,4
Captain	0	0	0	0	0
Lieutenant	0	0	0	0	0
Sergeant	0	0	0	0	0
Miner	0	0	0	0	0
Scout	0	0	0	0	0
Spy	0	0,25	0,5	0,75	1
Bomb	0	0,25	0,5	0,75	1

If the detection factor = 0 for all ranks, the coefficients are:

	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Marshal	-57634	8277	10281	11601	11785	9809	8432	10119	17438	2242	0
General	0	-50778	9988	12671	14027	11979	9438	10660	17004	3877	0
Colonel	0	0	-86688	21437	22546	18438	15507	17273	25845	5823	0
Major	0	0	0	-116446	33407	25714	20248	19965	31809	4894	0
Captain	0	0	0	0	-146744	39357	28033	22272	45428	3367	0
Lieutenant	0	0	0	0	0	-125854	28778	20913	47094	2249	0
Sergeant	0	0	0	0	0	0	-92213	25979	38015	1718	0
Miner	0	0	0	0	0	0	0	-102993	30550	2755	48813
Scout	0	0	0	0	0	0	0	0	-133225	7052	0
Spy	9010	0	0	0	0	0	0	0	0	-12489	0
Bomb	3104	4294	6965	8891	11802	11254	8212	0	33092	1699	-106403

At a detection factor = 0,1 all ranks become:

	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Marshal	-57634	8277	10281	11601	11785	9809	8432	10119	17438	3143	293,2
General	552,2	-50778	9988	12671	14027	11979	9438	10660	17004	3877	412,2
Colonel	632,6	590,6	-86688	21437	22546	18438	15507	17273	25845	5823	670,6
Major	622,2	667,3	1382,1	-116446	33407	25714	20248	19965	31809	4894	850,3
Captain	550,8	667	1340,2	2412,8	-146744	39357	28033	22272	45428	3367	1135,9
Lieutenant	441,7	535,9	1015,3	1707,1	3120,7	-125854	28778	20913	47094	2249	1093,9
Sergeant	360,9	377,1	753,8	1207,3	2039,9	2432,6	-92213	25979	38015	1718	793,1
Miner	339,3	290,1	636,2	977	1399,3	1598,6	2342,8	-102993	30550	2755	48813
Scout	1109,2	1008,6	1694,1	2296,2	3609,8	4088,9	3507,6	2769,2	-133225	7052	3192,6
Spy	9025,9	98,7	247,8	264	207	157,9	131,7	235	237,4	-12489	160,2
Bomb	3104	4294	6965	8891	11802	11254	8212	3612,9	33092	1699	-106403

And for a detection factor = 0,2 the 11 by 11 matrix is:

	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	Bomb
Marshal	-57634	8277	10281	11601	11785	9809	8432	10119	17438	4044	586,4
General	1104,4	-50778	9988	12671	14027	11979	9438	10660	17004	3877	824,4
Colonel	1265,2	1181,2	-86688	21437	22546	18438	15507	17273	25845	5823	1341,2
Major	1244,4	1334,6	2764,2	-116446	33407	25714	20248	19965	31809	4894	1700,6
Captain	1101,6	1334	2680,4	4825,6	-146744	39357	28033	22272	45428	3367	2271,8
Lieutenant	883,4	1071,8	2030,6	3414,2	6241,4	-125854	28778	20913	47094	2249	2187,8
Sergeant	721,8	754,2	1507,6	2414,6	4079,8	4865,2	-92213	25979	38015	1718	1586,2
Miner	678,6	580,2	1272,4	1954	2798,6	3197,2	4685,6	-102993	30550	2755	48813
Scout	2218,4	2017,2	3388,2	4592,4	7219,6	8177,8	7015,2	5538,4	-133225	7052	6385,2
Spy	9041,8	197,4	495,6	528	414	315,8	263,4	470	474,8	-12489	320,4
Bomb	3104	4294	6965	8891	11802	11254	8212	7225,8	33092	1699	-106403

Etcetera.

See appendix A for methods to solve sets of linear equations.

3 Findings and conclusions

Starting point for this study was actually the intuitive assumption that the value of pieces and ranks can be derived from the material gain that ranks achieve on the average in games. That was the motive to gather data about material gain by captures in games. Subsequently ways to convert data to values were investigated.

In this document some models are presented. In these models the assumption of a detection factor appears to be crucial in order to get more or less acceptable values for ranks. Another assumption is that only pieces involved in a capture should be counted. That assumption looks somewhat artificial, a more plausible assumption is that all pieces in an initial setup should be counted.

Other models with various assumptions have been examined. For example the more plausible assumption that all pieces in the start position of a game should be counted, has been examined, but this produced absurdly high values for bomb and miner. The results of most other time consuming attempts were disappointing. In most cases absurd rank values (for example negative values) were produced without the ratios and coherence of the here presented models.

This all suggests that the results of this study have the character of a fluke. Results like these do not constitute hard evidence for the idea that values of ranks stem from their capability to generate gain by the capture of material and (in case of a loss) by the detection of ranks in Stratego. But anyhow this is a start, perhaps a step in the right direction.

For those who want to apply static values of pieces and ranks in Stratego, this study might be a source of inspiration. It is conceivable that in other games like Stratego the static value of pieces can be determined a similar way too.

Appendix A How to solve a system of linear equations

A.1. Theory

Long ago C.F. Gauss (1777-1855) described how systems of linear equations can be solved.
In the majority of books about linear algebra, descriptions of the solving methods can be found.
Systems of equations with 0 at one side of the = sign are a special case:

- They have the trivial solution $v^i = 0$ for each i.
- In addition there are infinite solutions that have in common that the values v^i have a fixed ratio to each other.

A.2 A solving method

With an example in this paragraph is explained how a system of solving 3 linear equations with 3 variables works.

0 = -x + y + z   Equation 1
0 = x – 2y +z   Equation 2
0 = x +y -3z   Equation 3

Or in matrix format:

	x	y	z
x	-1	1	1
y	1	-2	1
z	1	1	-3

Add Equation 1 to Equation 2 and 3.
0 = -x + y + z   Equation 1
0 = 0x – y +2z   Equation 2
0 = 0x +2y -2z   Equation 3

Or in matrix format:

	x	y	z
x	-1	1	1
y	0	-1	2
z	0	2	-2

In equation 2 y = 2z. Substitute a random value for z, for example 1, and equation 2 produces a value 2 for y. Subsequently equation 1 produces a value 3 for x.
A value 2 for z would have given a value 4 for y and a value 6 for x: the mutual ratios stay the same.

Totally analogue this method has been applied to the systems of 11 equations with 11 variables.

Appendix B Statistics of games and moves

It was necessary to construct a table with intermediate results for the computation of rank values. That offers the opportunity to show data about games and moves prior to the data about rank values.

B.1 Games

In this study data about 29397 games have been used.

At first an overview of numbers of games for Red and Blue.

	Number of games
Red wins	13799
Blue wins	14551
Draw	1047

In more detail:

Winner	By	Number of games
Red	All pieces captured	1581
	Flag captured	4738
	Time out	144
	Time out clock	41
	Surrender	7295
Blue	All pieces captured	1696
	Flag captured	4923
	Time out	117
	Time out clock	41
	Surrender	7774
Draw	Players have left the gam	447
	Time out clock	3
	No movable piece left	6
	Draw given	591

About 30 percent of the result in games is decided by capture of the flag.
Then it may be interesting to know how often ranks win a game by capture of the flag too:

	Number of flag captures
Marshal	447
General	475
Colonel	793
Major	792
Captain	682
Lieutenant	408
Sergeant	330
Miner	4177
Scout	1364
Spy	193
Total	9661
Number of Games	29397

B.2 Moves

In this study data about 29397 games and 9783063 moves have been used.

Overview of attacks and moves to empty squares per rank								
Number					Percentage			
Rank	To empty squares	Attacks	All moves		Rank	To empty squares	Attacks	All moves
Marshal	797315	87948	885263		Marshal	90	10	10
General	651350	92283	743633		General	88	12	9
Colonel	1029570	133627	1163197		Colonel	89	11	13
Major	1097178	140624	1237802		Major	89	11	14
Captain	946438	143133	1089571		Captain	87	13	13
Lieutenant	654758	110001	764759		Lieutenant	86	14	9
Sergeant	484538	80774	565312		Sergeant	86	14	7
Miner	928143	102850	1030993		Miner	90	10	12
Scout	595018	251966	846984		Scout	70	30	10
Spy	274379	18982	293361		Spy	94	6	3
Total	7458687	1162188	8620875		Average	87	13	100

The high percentage moves to an empty square makes Stratego unfit for the implementation of the minimax algorithm. Only if paths from piece to piece are combined into 1 move (a jump move) then maybe the minimax algorithm may get a chance of success.

Interesting is the percentage human mistakes by a losing attack on an already known piece.

Attacking rank	Number of attacks having a loss to a known piece											
											Total	Percentage
	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	172	87948	0
Marshal	147										172	92283	0
General	169	276									259	133627	1
Colonel	192	285	615								388	140624	1
Major	206	310	591	901							443	143133	2
Captain	214	319	583	864	973						315	110001	3
Lieutenant	235	329	577	785	899	619					281	80774	5
Sergeant	209	315	570	671	720	548	348					102850	3
Miner	1078	939	1759	2362	2687	1800	1051	659			1166	251966	5
Scout		170	280	294	196	132	76	78	475		97	18982	9
Spy													
Bomb	2450	2943	4975	5877	5475	3099	1475	737	475	0		1162188	2

Interesting too is the percentage blind attacks with a loss to unknown pieces.

Attacking rank	Number of losing gambling attacks to an unknown piece											
											Total	Percentage
	Marshal	General	Colonel	Major	Captain	Lieutenant	Sergeant	Miner	Scout	Spy	2932	10777	27
Marshal	4										4122	15748	26
General	16	39									6706	22343	30
Colonel	80	86	202								8503	25896	34
Major	185	244	470	1021							11359	36601	36
Captain	223	381	650	1333	3249						10939	37998	44
Lieutenant	168	265	498	1128	2636	3711					7931	28030	58
Sergeant	210	215	622	1514	2571	3173	4498					52148	25
Miner	1073	1367	3023	6435	14335	17304	15727	10989			31926	131419	78
Scout		12	52	156	289	298	297	530	298		1602	3824	92
Spy													
Bomb	1959	2609	5517	11587	23080	24486	20522	11519	298	0		364784	28

The probability of a loss is 30 percent or less for top ranks. In case of a material disadvantage with a chance of game loss it will be a promising approach to capture blindly a few pieces in order to regain material.